Unit 4
DECISION ANALYSIS

Lesson 35

Learning Objective:

- **Illustrate the models of decision making under conditions of uncertainity.**

Hello students,

In previous lesson you learned three criterions for decision under uncertainity.

Now,
In this lesson you will study about the other three criterion used for taking **decision under uncertainity.**

The fourth criterion is:

SAVAGE MINIMAX REGRET CRITERION

The Minimax Regret criterion focuses on avoiding regrets that may result from making a non-optimal decision. Although regret is a subjective emotional state, the assumption is made that it is quantifiable in direct (linear) relation to the rewards of the payoff matrix.

Regret is defined as the opportunity loss to the decision maker if action alternative A_i is chosen and state of nature S_j happens to occur. **Opportunity loss** is the payoff difference between the best possible outcome under S_j and the actual outcome resulting from choosing A_i. Formally:

OL_{ij} = (row j maximum payoff) - R_{ij} for positive-flow payoffs (profits, income)

OL_{ij} = R_{ij} - (row j minimum payoff) for negative-flow payoffs (costs, losses)

where R_{ij} is the reward value (payoff) for column i and row j of the payoff matrix R.

Note that opportunity losses are defined as nonnegative numbers. The best possible OL is zero (no regret) and the higher the OL value, the greater the regret.

Savage's Minimax Regret decision rule

1. Convert the payoff matrix R = $\{R_{ij}\}$ into an opportunity loss matrix OL = $\{OL_{ij}\}$.
2. Apply the minimax rule to the OL matrix.

Let's assume that ACME's managers have decided to analyze the problem using opportunity losses instead of the monetary payoffs.

First they must derive the OL table from the payoff matrix R.

STATES OF NATURE	ALTERNATIVES				
	Large plant	Just Right plant	Small plant	No plant	Best
High demand	15	9	3	0	15
Medium demand	3	4	2	0	4
Low demand	-6	-2	1	0	1

Note,
Row max = best possible outcome for that particular S_j

The OL table can now be obtained by subtracting each entry R_{ij} from its column's best payoff. The minimax rule is then applied to the OL (regret) table:

STATES OF NATURE	ALTERNATIVES			
	Large plant	Just Right plant	Small plant	No plant
High demand	0	6	12	15
Medium demand	1	0	2	4
Low demand	7	3	0	1
Minimax Regret	7	6 Minimax	12	15

Decision is "*Just Right*"

Let us find out what is the **Economic Interpretation of Opportunity Losses**
OL values consist of two components: actual monetary losses (if any) and unrealized potential profits. Consider the OL for *JR* x *H*, which is 6. ACME would stand to make $9 million (R matrix) if they choose plant *JR* and market state *H* occurs. So there would be no monetary loss. Still, by choosing *JR* ACME's managers would forgo the opportunity to gain an additional $6 million — assuming state *H* actually occurs. If state *H* does indeed happen, ACME's managers would not feel entirely satisfied: there would be an element of regret present for not having made the "correct" decision, which was plant *L*. This regret is assumed to be equal to the lost opportunity: $6 million.

Consider now the OL value for *JR* x *W*, which is 3. The actual payoff would be a $2 million loss. But in addition there would be a regret factor for not capitalizing on the $1 million profit ACME could have had IF they had chosen plant *S*.

Finally, notice that the OL for *S* x *W* is zero because that was the right decision for that particular state of nature: there is no actual monetary loss and no potential profits were forgone. Thus, no regrets.

Note that for every column in OL there must be at least one entry $OL_{ij} = 0$ (that is, at least one "best" outcome for each state of nature). This is not necessarily true for every row.

It should be clear that standard accounting information is incomplete in the sense that OL values are neither recorded nor obtainable ex post facto. Accounting rules state that a journal entry is performed only if a transaction

actually occurs. Consequently, the potential benefits of alternate decision strategies cannot be determined from financial accounting statements.

Now Let's see what are the drawbacks of this criterion:

Critique of Minimax Regret Criterion

Minimax Regret is a better decision criterion than Maximax or Maximin and, arguably, Hurwicz as well. Although it employs the far-from-robust minimax logic, the values over which it operates (opportunity losses) contain more problem information (actual monetary losses plus unrealized potential profits), leading to a more informed decision than was possible with any of the three previous models. Nevertheless, it still fails to employ all of the available problem information and is therefore not a rationally acceptable criterion.

Minimax Regret is a conservative criterion, as is Maximin/Minimax. However, it is not as extreme in its pessimism as the latter. Note that in ACME's decision problem, Minimax Regret recommended a different (middle-of-the-road) decision alternative than Maximin.

There is no guarantee this will always be so, but it does show that minimaxing regrets is not as conservative an approach as maximining positive-flow payoffs.

Another important criterion is:

LAPLACE INSUFFICIENT REASON CRITERION

The Laplace criterion is the first to make use of explicit probability assessments regarding the likelihood of occurrence of the states of nature. As a result, it is the first elementary model to use all of the available information in the payoff matrix.

The Laplace argument makes use of Johann Bernoulli's Principle of Insufficient Reason.

To begin with, Laplace posits that to deal with uncertainty rationally, probability theory must be invoked.

This means that for each state of nature S_j in S, you (the decision maker) should assess the probability p_j that S_j will occur.

Now the **Principle of Insufficient Reason** states that if no probabilities have been assigned by you (assumed to be rational and capable of handling basic probability theory), then it follows there was insufficient reason for you to indicate that any one state S_j was more or less likely to occur than any other state.
(I feel a rational decision maker, would assign a probability distribution to S as a matter of course.)

Consequently, all the states S_j must be equally likely. Therefore, the probability p_j for every S_j must be $1/n$, where n is the number of states of nature in S.

Pretty neat logic! We'll check it out in the critique.

Laplace decision rule
1. Assign $p_j = p(S_j) = 1/n$ to each S_j in S, for $j = 1, 2, \ldots, n$.
2. For each A_i (payoff matrix row), compute its expected value:
$$E(A_i) = \Sigma_j \; p_j \; (R_{ij}).$$
3. Select the action alternative with the best E (A_i) as the chosen decision.
"Best" means max for positive-flow payoffs (profits, revenues) and min for negative-flow payoffs (costs, losses).

Let's assume that ACME's managers believe all three market states (H, M, W) to be equally probable. Then *and only then* is the use of the Laplace criterion warranted.

Here according to Laplace decision each state of nature occurs with a probability 1/3.

STATES OF NATURE	ALTERNATIVES			
	Large plant	Just Right plant	Small plant	No plant
High demand	15	9	3	0
Medium demand	3	4	2	0
Low demand	-6	-2	1	0
E (A_i)	(15+3-6)/3 = **4**	(9+4-2)/3 = 3.67	(3+2+1)/3 = 2	(0+0+0)/3 = 0

The decision comes out to choose “to build a large plant”.

Now Let’s see what are the drawbacks of this criterion:

Critique of Laplace Insufficient Reason Criterion

By assigning a (uniform) probability distribution to S, Laplace is able to take into account all of the available information in R and is therefore a rationally acceptable decision criterion *assuming the states of nature S_j are indeed uniformly distributed* (that is to say, are equally probable).

There is nothing intrinsically wrong with the Laplace criterion but there is a danger of improperly using it when the states of nature are not in fact equally probable. We shall see this again in our discussion of assessing probabilities subjectively.

The weakness in the argument posed by the Principle of Insufficient Reason is the implicit assumption that all decision makers who have not assigned a probability distribution to S have not done so because there is no reason to believe the states are not equally probable. Real people can and commonly do depart from the idealized paradigm of the rational decision maker, and may well not assess probabilities quantitatively because they are not accustomed to do so.

Now we move on to last but not the least criterion of decision making under uncertainity.

MAXIMUM LIKELIHOOD (MODAL) CRITERION

This criterion considers only the event (state of nature) most likely to occur as the basis for the decision, excluding all other events from consideration. Maximum likelihood is a widely used statistical decision rule employed in many scientific and technical applications, usually in conjunction with other quantitative methods. It is also often used informally in personal decision making by non-specialists, usually by itself.

This latter usage, commonly called the *modal* criterion, can lead to improper reasoning, flawed problem analysis and poor decisions. The term "modal" refers to the *mode* of a statistical distribution.

Let's assume that ACME's managers, perhaps because of habit, were inclined to use the modal decision criterion. They would then act in the following manner:

Maximum likelihood (modal) decision rule

1. Select the state S_j most likely to occur. This can be done qualitatively by judgment or intuition.
2. Exclude from further consideration the remaining states of nature in S.
3. Determine the best payoff (max for positive flows, min for negative flows) in the chosen column (S_j). The action alternative A_i corresponding to this payoff is the chosen decision.

Use ACME's decision matrix defined previously and assume state *Medium demand* as the most likely:

STATES OF NATURE	ALTERNATIVES			
	Large plant	Just Right plant	Small plant	No plant
High demand	15	9	3	0
Medium demand	3	4	2	0
Low demand	-6	-2	1	0

This would yield

STATES OF NATURE	ALTERNATIVES			
	Large plant	Just Right plant	Small plant	No plant
Medium demand	3	**4**	2	0

Decision is : Just Right plant

Now Let's see what are the drawbacks of this criterion:

Critique of Maximum Likelihood (Modal) Criterion

We have been criticizing all decision rules that leave out available problem information as irrational, and here comes the modal criterion that does precisely that. In real life, however, things are usually a bit more complicated and irrationality may not be as obvious. The payoffs in the decision matrix are not, for the most part, given a priori: they represent data that must be collected. This means time and effort (and hence, a cost) must be expended in collecting the data. Some decision makers may see no point in collecting payoff data for states of nature deemed unlikely to occur from the modal perspective. So the decision matrix they develop may actually be just the column vector for the modal state. The danger of this approach should be clear: decisions are being made from a position of ignorance.

Decide for yourself

Consider the following table:

Probability p_j	STATES OF NATURE	ALTERNATIVES	
		A1	A2
0.4	S1	1	0
0.2	S2	0	100
0.2	S3	0	100
0.2	S4	0	100

The modal decision for the matrix below is A1. Would you forgo a 60% chance of getting $100 just because the mode points to a $1 gain?

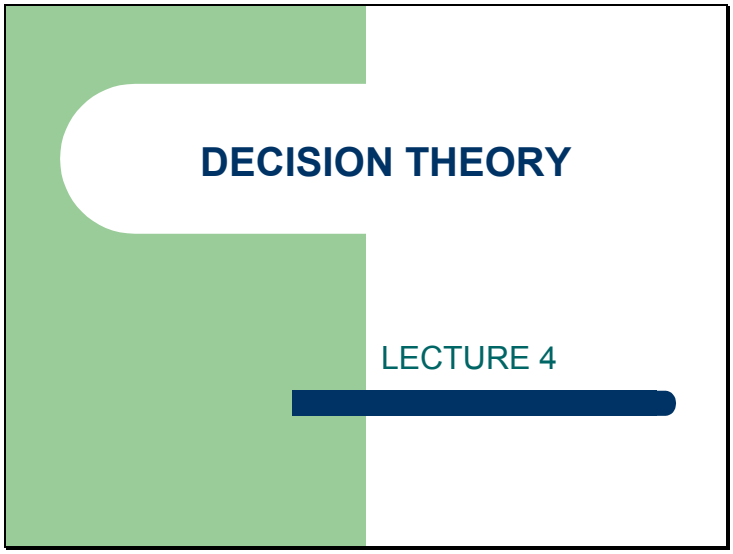
So, now let us summarize today's discussion:

Summary

We have discussed in details about **Decision making under uncertainity**.

- The Savage Minimax Regret Criterion.
- The Laplace Insufficient Reason Criterion
- The Maximum Likelihood (Modal) Criterion.

Slide 1



Slide 2

THE SAVAGE MINIMAX REGRET CRITERION

Savage's Minimax Regret decision rule

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Slide 3

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Laplace decision rule

1. Assign $p_j = p(S_j) = 1/n$ to each S_j in S, for $j = 1, 2, ..., n$.
2. For each A_i (payoff matrix row), compute its expected value: $E(A_i) = \Sigma_j \; p_j (R_{ij})$.
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Slide 4

THE MAXIMUM LIKELIHOOD (MODAL) CRITERION

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2. Exclude from further consideration the remaining states of nature in S.
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4. The action alternative A_i corresponding to this payoff is the chosen decision.